# FAIRER SERVICE EXCHANGE MECHANISMS FOR TENNIS WHEN SOME PSYCHOLOGICAL FACTORS EXIST 

Pollard, G. ${ }^{1}$ and Barnett, T. ${ }^{2}$<br>${ }^{1}$ School of Information Sciences and Engineering, University of Canberra, Australia<br>${ }^{2}$ Faculty of Life and Social Sciences, Swinburne University of Technology, Australia


#### Abstract

In a tennis match it is not uncommon for games to 'go with service' (ie 1-0, 1-1, 2-1, 2-2, $3-2, \ldots)$. When this occurs, the player who serves first is either ahead by one game, or the games' score is equal. Some commentators, players,...argue that the person who serves first has a psychological advantage in that his/her opponent is very often 'playing catch-up'. Assuming that such a (non-zero) psychological advantage of 'being ahead in the games' score' exists, the advantage of serving first in a set between two equal players, is determined. In the presence of such 'front-runner' psychological effects, alternative methods or rules for allocating service to the players are considered, and some are shown to be fairer than the present rule. A proposal consisting of two modifications to the present rules is put forward for consideration. One of these modifications is very easy to apply.

The reverse psychological effect to the above, the 'back-to-the-wall' effect, occurs when a player performs better when behind. The proposal is seen to be fairer than the present method for the cases in which both player A has either a positive or negative psychological effect and player B also has an equivalent positive or negative effect. Further, the application of the proposal to doubles is also considered and a modification for doubles suggested for consideration.


## KEY WORDS

rules in tennis, psychological advantage in tennis, back-to-the-wall effect in tennis, improved fairness in cricket

## INTRODUCTION

Many people believe that the person who serves first in a set of tennis has an advantage. This is because games often 'go with service', so that the first server is quite often ahead on the games' score, giving that player a psychological advantage. In this paper the extent of this advantage is analysed, by considering two identical players.

A fair scoring system has the characteristic that in a match between two equal players, each player has a probability of 0.5 of winning. A scoring system that does not have this characteristic is unfair.

Several methods for attempting to overcome the advantage noted above, are considered in Section 2 of this paper, and two methods that reduce the advantage are proposed.

Consideration is then given to the case of the reverse psychological effect, the 'back-to-thewall' effect. The performance of the two proposed methods is then analysed in the presence of both psychological effects.

Finally, the case of doubles is analysed. With four players on the court, there is scope for considering additional methods for overcoming the advantage of serving first. Minor changes to the above two methods are seen to apply.

Earlier studies have considered the advantage gained by lifting play in certain circumstances (Morris, 1977; Pollard, 2002). There is however very little reported empirical evidence of psychological advantages in tennis. A 'first game effect' in a match, namely that fewer breaks occur in the first game of the match, has been identified (Magnus and Klaasen, 1999). More recently, in men's singles grand slam tennis, the better player in a match has been shown to possess a 'back-to-the-wall' effect (Pollard, Cross and Meyer, 2006).

## METHODS

## (a) Singles

It is assumed that player A has a probability PA of winning a game on service when the games' scores are equal (ie $0-0,1-1,2-2, \ldots$ ), that it is PA+D when he/she is ahead in the games' score, and that it is PA-D when he/she is behind. Correspondingly, it is assumed that player B has a probability PB of winning a game on service when the games' scores are equal, that it is $\mathrm{PB}+\mathrm{D}$ when he/she is ahead in the games' scores, and PB-D when behind. Thus, the psychological advantage of being ahead might be called the 'frontrunner' effect, and is represented by D.
(b) A set of singles

Firstly we consider a tiebreak set of tennis between two equal players $(\mathrm{PA}=\mathrm{PB})$ with equal psychological factors, D. For simplicity, it is assumed throughout this paper that the two equal players have an equal chance of winning the tiebreak game if it is played (at 6-6). Assuming player A serves in the first game of the set, the probability player A wins the set can be evaluated using a branching diagram or using recurrence methods. For example, when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$, the probability that the games' score reaches $2-0,1-1$ and $0-2$ is $(0.6)(0.5)=0.30,(0.6)(0.5)+(0.4)(0.3)=0.42$ and $(0.4)(0.7)=0.28$ respectively. Further, the probability the games' score reaches $3-0,2-1,1-2$ and $0-3$ is $(0.30)(0.7)=$ $0.210,(0.30)(0.3)+(0.42)(0.6)=0.342,(0.42)(0.4)+(0.28)(0.5)=0.308$ and $(0.28)(0.5)=$ 0.140 respectively. Continuing in this manner, and adding the probabilities that player A wins $6-0,6-1,6-2,6-3,6-4,7-5$ or $7-6$, it follows that the probability player A wins a tiebreak set is equal to 0.5164 (see Table 1).
(c) A match of the present best-of-three tiebreak sets

We now consider a match of the best-of-three tiebreak sets between two such equal players. We note that if player A serves in the first game of a set and the set lasts an even number of games (ie the set score is 6-0, 6-2, 6-4 or 7-5), then, under the present service exchange rules, player A also serves first in the next set (otherwise player B serves first). Thus, if player A has an advantage of serving first in the first set, he/she also has that advantage in
the second set when an even number of games is played in the first set. The probability that player A wins a tiebreak set in an even number of games, and the probability he/she wins the set in an odd number of games are given in Table 1. Corresponding probabilities are also given for player B.

Table 1: The probabilities two equal players A and B win a tiebreak set in an even and odd number of games when there is a probabilistic advantage D in being ahead in games'
scores.

| PA | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PB | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
| D | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| P(A wins even) | 0.2770 | 0.2642 | 0.2499 | 0.2342 | 0.2175 | 0.2001 | 0.1826 |
| P(A wins odd) | 0.2230 | 0.2438 | 0.2665 | 0.2913 | 0.3187 | 0.3490 | 0.3832 |
| P(A wins) | 0.5000 | 0.5080 | 0.5164 | 0.5256 | 0.5362 | 0.5491 | 0.5658 |
| P(B wins even) | 0.2770 | 0.2882 | 0.2976 | 0.3052 | 0.3103 | 0.3122 | 0.3089 |
| P(B wins odd) | 0.2230 | 0.2038 | 0.1860 | 0.1693 | 0.1535 | 0.1387 | 0.1253 |
| P(B wins) | 0.5000 | 0.4920 | 0.4836 | 0.4744 | 0.4638 | 0.4509 | 0.4342 |

Table 2 lists ten mutually exclusive outcomes for a best-of-three sets match won by player A, given player A serves in the first game of the first set. Note that in Table 2 a prefix is used to denote the server in the first game of the set, a capital letter is used to denote the winner of the set, and odd/even classifies the number of games played in the set. The fourth column in Table 2, headed probability, is obtained by multiplying the probabilities of the events in columns 1 and 2, or columns 1,2 and 3 for the case in which $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$. These probabilities can in turn be obtained from Table 1. Note that to obtain probabilities for sets with player B serving first, we can just reverse the roles of player A and player B (as they are equal players). Thus, for example, the probability of the event bBeven equals the probability of the event aAeven.

Table 2: The probabilities of ten mutually exclusive outcomes for a best-of-three tiebreak sets match won by player A , when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$.

| First set | Second set | Third set | Probability |
| :--- | :--- | :--- | :--- |
| aAeven | aAeven or odd |  | 0.1290 |
| aAodd | bAeven or odd |  | 0.1289 |
| aAeven | aBeven | aAeven or odd | 0.0384 |
|  |  |  |  |
| aAeven | aBodd | bAeven or odd | 0.0225 |
| aAodd | bBeven | bAeven or odd | 0.0322 |
| aAodd | bBodd | aAeven or odd | 0.0367 |
| aBeven | aAeven | aAeven or odd | 0.0384 |
| aBeven | aAodd | bAeven or odd | 0.0384 |
| aBodd | bAeven | bAeven or odd | 0.0268 |
| aBodd | bAodd | aAeven or odd | 0.0179 |
|  |  | TOTAL | 0.5091 |

It can be seen from Table 2 that when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$, the probability that player A wins a best-of-three tiebreak sets match given he/she serves first in the match, is equal to 0.5091 . Column (c) in Table 3 gives corresponding results for the other values of PA, PB and D in Table 1. Thus, it is clear that player A gains a match advantage by serving first in the first set.

Table 3: The probability player A wins a best-of-three tiebreak sets match when there is a probabilistic advantage D in being ahead in the games' score, and player A serves first in the match.

| PA | PB | D | (c) P(A wins) | (d) P(A wins) | (e) P(A wins) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5 | 0.5 | 0.1 | 0.5000 | 0.5000 | 0.5000 |
| 0.55 | 0.55 | 0.1 | 0.5045 | 0.5040 | 0.4996 |
| 0.6 | 0.6 | 0.1 | 0.5091 | 0.5082 | 0.4993 |
| 0.65 | 0.65 | 0.1 | 0.5140 | 0.5128 | 0.4991 |
| 0.7 | 0.7 | 0.1 | 0.5195 | 0.5182 | 0.4993 |
| 0.75 | 0.75 | 0.1 | 0.5259 | 0.5248 | 0.5002 |
| 0.8 | 0.8 | 0.1 | 0.5340 | 0.5335 | 0.5022 |

(d) An alternative best-of-three tiebreak sets system

We now consider the effect of modifying the service exchange mechanism 'across-sets'. The case in which service alternates at the beginning of each set is considered. It can be shown using values from Table 1 that, given player A serves first in the first set, player B first in the second set and player A first in the third set (if necessary), the probability that player A wins the match is equal to 0.5082 when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$. This is a slight improvement on the present situation analysed in (c) above. The corresponding probability values for other values of PA, PB and D are given in column (d) of Table 3. Also, it can be seen that if we modify this service exchange mechanism so that player B serves first in both the second and third sets (if necessary), player A's probability of winning the match is now $1-0.5082$ when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$. Thus, this modification to the third set server leads to no overall difference in fairness on simply alternating service at the beginning of each set. Also, note that when a set lasts an even number of games under this system, the same person serves the last game in that set and the first game of the following set (if played). This should not be a problem as the players have a two minute rest between sets.
(e) An alternative 'across-sets' service exchange mechanism

We now consider a slight variation in the third set to the service exchange mechanism considered in (d) above. We suppose the server in the third set is determined as at present. That is, given player B served first in the second set, player A serves first in the third set if there is an odd number of games in the second set, and player B serves first if there is an even number of games. It can be shown using values from Table 1 that, with this variation to (d) above, the probability player A wins the match is equal to 0.4993 when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$. This represents a considerable improvement on the situation analysed in (d) above. The corresponding probability values for other values of PA and PB are given in column (e) of Table 3. (Another variation in the third set to this service exchange mechanism could be where the server in the third set is the player who won the most
number of games in the first two sets. If this countback procedure leads to a tie in the number of games won, we use the present service exchange mechanism as in (c) above. This two stage countback mechanism in fact leads to a very small increase in fairness, but this is not considered to be worthy of further discussion. Other countback methods have been considered in another context (Pollard and Noble, 2006).)
(f) A 'within-set' service exchange mechanism

A service exchange mechanism similar to that used in the tiebreak game is considered. Player A serves in the first game, player B serves in the next two games, player A serves in the following two games, ...(ie $A, B, B, A, A, B, B, A, A, B, B, A)$. The present stopping rules ( 6 -$0,6-1,6-2, \ldots 7-5$ ) are used and the tiebreak game is played if the games' score reaches 6-6. Under this service exchange mechanism, assuming that the two equal players have an equal chance of winning the tiebreak game if played, the probability player A wins the set is equal to 0.5046 when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$. It can be seen by comparing Table 4 column 4 with Table 1 that this service exchange mechanism within a set considerably reduces the advantage that player A obtains by serving first. A major disadvantage of this mechanism is that player B is required to serve two games in a row on (up to) three occasions, and player A is required to do the same on (up to) two occasions. This mechanism is not considered to be of particular practical relevance. However, if this mechanism was used, change-of-ends might occur after an even number of games is played, so that, when a player serves two games in a row, they are from different ends of the court with a time-break between those service games.

Table 4: The probability player A wins a tiebreak set, given player A serves first and the service-game order is $A, B, B, A, A, B, B, A, A, B, B, A$ (column 4) and when the service order is $\mathrm{A}, \mathrm{B}, \mathrm{B}, \mathrm{A} ; \mathrm{B}, \mathrm{A}, \mathrm{A}, \mathrm{B} ; \mathrm{B}, \mathrm{A}, \mathrm{A}, \mathrm{B}$ (column 5).

| PA | PB | D | Col4 | Col5 |
| :--- | :--- | :--- | :--- | :--- |
| 0.5 | 0.5 | 0.1 | 0.5000 | 0.5000 |
| 0.55 | 0.55 | 0.1 | 0.5023 | 0.5017 |
| 0.6 | 0.6 | 0.1 | 0.5046 | 0.5032 |
| 0.65 | 0.65 | 0.1 | 0.5069 | 0.5046 |
| 0.7 | 0.7 | 0.1 | 0.5092 | 0.5056 |
| 0.75 | 0.75 | 0.1 | 0.5116 | 0.5062 |
| 0.8 | 0.8 | 0.1 | 0.5141 | 0.5060 |

(g) Another 'within-set' service exchange mechanism

A variation of the 'tiebreak-like' service exchange mechanism in the above paragraph is the following-A,B,B,A; B,A,A,B;B,A,A,B. Using the present stopping rules and this mechanism, it can be shown that the probability player A wins a set is equal to 0.5032 when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$ (see Table 4 column5). It can be seen from Table 4 that this mechanism gives a slight improvement on that in the previous paragraph. Players A and B would each have to serve two games in a row on (up to) two occasions, and change-of-ends could again be 'on-the-even'. This mechanism is also considered to be of little practical relevance.
(h) A third 'within-set' service exchange mechanism

A further 'within-set' mechanism is now considered. Suppose player B, the server in the second game of the set, is allowed to serve two games in a row on (up to) one occasion in the set (whilst player A never serves two games in a row). The possibilities for the (maximum of) twelve games in a set (up to 6-6) are
(i) $A, B ; B, A, B, A, B, A, B, A, B, A$
(ii) $\mathrm{A}, \mathrm{B}, \mathrm{A}, \mathrm{B} ; \mathrm{B}, \mathrm{A}, \mathrm{B}, \mathrm{A}, \mathrm{B}, \mathrm{A}, \mathrm{B}, \mathrm{A}$
(iii) $\mathrm{A}, \mathrm{B}, \mathrm{A}, \mathrm{B}, \mathrm{A}, \mathrm{B} ; \mathrm{B}, \mathrm{A}, \mathrm{B}, \mathrm{A}, \mathrm{B}, \mathrm{A}$
(iv) $A, B, A, B, A, B, A, B ; B, A, B, A$
(v) $A, B, A, B, A, B, A, B, A, B ; B, A$

For these five alternatives it can be shown that the probability player A wins the set when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$ is
(i) 0.4976
(ii) 0.5037
(iii) 0.5078
(iv) 0.5111 and
(v) 0.5140 (see Table 5)

Thus, the mathematics suggests that it would be in player B's interest to elect to serve the two games in a row early (rather than later) in the set. However, he/she might prefer to elect to do it later in the set when the games are more important, or alternatively just after having played an 'easy' service game. It would seem that such a system would increase the 'excitement' of the set. 'Change-of-ends' might again be 'on-the-even'.

The player who serves first in a match against an equal opponent has been shown to have an overall advantage in the situation in which each player has the same psychological advantage when ahead in games' score within the set. Several methods of decreasing this advantage have been considered, and two of them would seem appropriate for consideration. Firstly, if service alternates at the beginning of each set (except the final third or fifth set), the benefit a player receives from serving first in the match is reduced. Secondly, if the player who serves second in a set is allowed to serve on two consecutive occasions within that set, the benefit the player receives from serving first in the set is reduced.

Table 5: The probability player A wins a tiebreak set for each of the five service order cases (i) to (v) in Section 2(h).

| PA | PB | D | (i) | (ii) | (iii) | (iv) | (v) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5 | 0.5 | 0.1 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |
| 0.55 | 0.55 | 0.1 | 0.4989 | 0.5018 | 0.5038 | 0.5054 | 0.5068 |
| 0.6 | 0.6 | 0.1 | 0.4976 | 0.5037 | 0.5078 | 0.5111 | 0.5140 |
| 0.65 | 0.65 | 0.1 | 0.4959 | 0.5059 | 0.5124 | 0.5174 | 0.5220 |
| 0.7 | 0.7 | 0.1 | 0.4935 | 0.5083 | 0.5178 | 0.5250 | 0.5313 |
| 0.75 | 0.75 | 0.1 | 0.4898 | 0.5109 | 0.5245 | 0.5345 | 0.5427 |
| 0.8 | 0.8 | 0.1 | 0.4840 | 0.5136 | 0.5330 | 0.5469 | 0.5576 |

(i) Another psychological factor

It has been argued by some players, commentators, spectators,...that some players possess a different psychological factor called the 'back-to-the-wall' effect. In this case the player is assumed to have a higher probability of winning a game when behind. We firstly consider the case in which both players possess this factor. Thus, player A is assumed to have a probability PA of winning a game on service when the games' scores are equal, and that it is PA+D when he/she is behind and that it is PA-D when ahead. Correspondingly, it is assumed that player B has a probability PB of winning a game on service when the games’ scores are equal, and that it is $\mathrm{PB}+\mathrm{D}$ when he/she is behind and that it is $\mathrm{PB}-\mathrm{D}$ when ahead. Thus, the psychological advantage of being behind is represented by D. Similarly to (b) above, and assuming player A serves first in the set, the probability player A wins a tiebreak set is 0.4806 (refer to Table 6) and the probability player A wins a best-of-three tiebreak sets match is 0.4896 (refer to Table 7 column (c) ) when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$. Using the 'across-sets' service exchange mechanism described in (e) above, player A's probability of winning such a modified best-of-three tiebreak sets match is 0.5007 when $\mathrm{PA}=\mathrm{PB}=0.6$ and $\mathrm{D}=0.1$ (refer to Table 7 column (d)). This represents a considerable improvement on the number immediately above.

Table 6: The probabilities two equal players A and B win a tiebreak set in an even and odd number of games when there is a probabilistic advantage D in being behind in games' scores.

| PA | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PB | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 |
| D | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| P(A wins even) | 0.2712 | 0.2502 | 0.2269 | 0.2012 | 0.1726 | 0.1405 | 0.1035 |
| P(A wins odd) | 0.2288 | 0.2403 | 0.2537 | 0.2685 | 0.2845 | 0.3013 | 0.3184 |
| P(A wins) | 0.5000 | 0.4905 | 0.4806 | 0.4697 | 0.4571 | 0.4417 | 0.4219 |
| P(B wins even) | 0.2712 | 0.2901 | 0.3066 | 0.3206 | 0.3315 | 0.3382 | 0.3384 |
| P(B wins odd) | 0.2288 | 0.2194 | 0.2128 | 0.2097 | 0.2114 | 0.2200 | 0.2397 |
| P(B wins) | 0.5000 | 0.5095 | 0.5194 | 0.5303 | 0.5429 | 0.5583 | 0.5781 |

Table 7: The probability player A wins a best-of-three tiebreak sets match when there is a probabilistic advantage D in being behind in the games' score, and player A serves first in the match.

| PA | PB | D | (c) P(A wins) | (d) P(A wins) |
| :--- | :--- | :--- | :--- | :--- |
| 0.5 | 0.5 | 0.1 | 0.5000 | 0.5000 |
| 0.55 | 0.55 | 0.1 | 0.4949 | 0.5004 |
| 0.6 | 0.6 | 0.1 | 0.4896 | 0.5007 |
| 0.65 | 0.65 | 0.1 | 0.4843 | 0.5008 |
| 0.7 | 0.7 | 0.1 | 0.4788 | 0.5006 |
| 0.75 | 0.75 | 0.1 | 0.4730 | 0.4997 |
| 0.8 | 0.8 | 0.1 | 0.4669 | 0.4974 |

The 'within-set' modifications considered in (f), (g) and (h) all decrease player B's probability of winning a set from $(1-0.4806)=0.5194$. The reason for this is that under these modifications player B is required to serve (on average) earlier in the match so he/she is less often behind (when his/her p-values are higher). The mechanisms in (f) and (g) were considered to be of little practical relevance. With respect to the mechanism in (h), as player B's probability of winning the set is decreased for all cases (i) to (v), player B would presumably not elect to serve two games in a row as he/she would only decrease his/her probability of winning the set.
(j) The combination of the two psychological factors

We now assume that player A possesses a 'front-runner' factor D1, and player B possesses a 'back-to-the-wall' factor D2. Player A's probability of winning a service game is equal to PA when the games' scores are equal, PA+D1-D2 when he /she is ahead in games' scores, and PA-D1+D2 when he/she is behind. Correspondingly, player B's probabilities on service are PB when equal, $\mathrm{PB}+\mathrm{D} 1-\mathrm{D} 2$ when B is ahead and $\mathrm{PB}-\mathrm{D} 1+\mathrm{D} 2$ when behind. It can be seen that player A's probability of winning a game on service is always PA when $\mathrm{D} 1=\mathrm{D} 2$, and player B's is always PB when D1=D2. Thus, the present scoring system is fair for this situation, as are the two recommendations in (h) above.
(k) Doubles

The situations for doubles are very similar. As an example, we consider section (b) above for the case in which $\mathrm{PA} 1=\mathrm{PB} 1=0.65$ and $\mathrm{PA} 2=\mathrm{PB} 2=0.55$ ( PA and PB both average 0.6 ), the only psychological factor being the 'front-runner' effect for every player and it is assumed to be $\mathrm{D}=0.1$, and the teams' chances at the tiebreak game are assumed to be equal.

When the service order is A1, B1, A2, B2,... ( the typical case in which each team uses their more effective server first) the probability team A wins the tiebreak set is 0.5189 . When the service order is A2, B2, A1, B1,...the probability team A wins the set is 0.5139 ( a fairer outcome than for the order in the previous sentence). This suggests a minor adjustment to the 'across-sets' modification in (e) above. Namely, if team A serves first in the first set (with service order A1, B1, A2, B2) and team B serves first in the second set (with service order B1, A1, B2, A2), then if team A serves first in the third set, the first two servers should be reversed (ie A2, B2, A1, B1,...), and if team B serves first in the third set, the order should be B2,A2,B1,A1.

Looking at the 'within-set' changes considered in section (h) (cases (i) and (ii) in particular), when the order is $\mathrm{A} 1, \mathrm{~B} 1 ; \mathrm{B} 2, \mathrm{~A} 2, \mathrm{~B} 1, \mathrm{~A} 1, \ldots$, the probability team A wins the set is 0.4820 , and when the order is $\mathrm{A} 1, \mathrm{~B} 1, \mathrm{~A} 2, \mathrm{~B} 2 ; \mathrm{B} 1, \mathrm{~A} 1, \mathrm{~B} 2, \mathrm{~A} 2, \ldots$, the probability team A wins the set is 0.5054 . This suggests that after four games have been played within a set, team B be allowed to play two service games in a row.

## RESULTS

Given two equal singles players with an equal psychological advantage when ahead, the player who serves first is shown to have a probability of winning the set greater than 0.5. His/her probability of winning a best-of-three sets match is also greater than 0.5 . Thus,
given the existence of a psychological advantage when ahead, the present best-of-three tiebreak sets scoring system is unfair.

It has been shown in the previous section that the present scoring system can be made fairer by two methods. Firstly, alternating service at the beginning of each set (with the server in the final third or fifth set being determined as at present) reduces the unfairness. Secondly, allowing the player who serves in the second game of a set to serve (only (up to) once) on two consecutive games within that set also reduces the unfairness.

The reverse psychological effect is when a player lifts his/her game when he/she is behind in games' score (the "back-to-the-wall" effect). The two methods above have also been shown to be applicable to the situation in which one player has the psychological advantage of being ahead or its reverse, whilst the other player also has this psychological advantage or its reverse.

Also for doubles, the above two methods were shown to decrease unfairness. Interestingly, the unfairness is further reduced by reversing the service order within each doubles pair for the final third or fifth set.

## DISCUSSION

The problem that the person or team that serves first in a set of tennis, has an advantage, has been long recognised. Indeed, it is an intrinsic difficulty within the tennis scoring system, and in this paper it has been quantified. The solution presently used is to toss a coin, so that each player or team has an equal chance of getting the advantage of serving first. A better solution is to modify the scoring system so that the advantage of serving first is decreased or reduced to zero. Scoring systems in which this advantage is zero have been devised (Miles, 1984), but their structures are quite different to the present tennis scoring system. A change to such a structure would be regarded by many people as a major change, and hence would be unlikely to gain acceptance. In this paper, minor changes to just the service exchange mechanism within the scoring system have been considered and shown to decrease the advantage gained by the person or team that serves first.

The methods of this paper can be used to analyse the one-day and test versions of a series of (say) three or five cricket matches. At present there is a toss before each match within the series. Assuming there is a psychological advantage in batting first in a match, then it can be shown that it is better to toss only before the first match within the series, and then alternate the first team to start the batting after that. The team to bat first in the final (third or fifth) match could be determined by some countback procedure. More generally, it can be seen that the toss of a coin is often used to create fairness in a situation that is intrinsically unfair. The irony of the situation in cricket is that the use of the toss of a coin three or five times only makes the rules about the first team to bat in each match not as good as they can be. One toss is not only enough, but it is better when followed by an alternating structure.

## CONCLUSIONS

For the situation in which players have a psychological advantage when ahead in games' score, the player who serves first in a set of tennis has been shown to have an advantage.

This (set) advantage can be decreased by alternating service at the beginning of each set (with the exception of the final third or fifth set which would be determined under the present rules). This change to the present service exchange mechanism would be very easy to implement, and it appears to have no real drawbacks.

The advantage of serving first in a set can be further reduced by allowing the player who serves in the second game of the set to serve on two consecutive games at some stage within that set. This might seem to be a little unusual at first, but it would appear to create some additional excitement in the set for the spectators. It would also create a strategic and additional dynamic element in the set for the players. However, it might be more difficult for such a change to gain acceptance.

Also, these two changes in the service exchange mechanism have been shown to be applicable when either or both players have either this psychological effect (the 'frontrunner' effect) or its reverse (the 'back-to- the-wall' effect).

Finally, the two changes have been shown to be applicable to doubles. The advantage of serving first in the match is further reduced in doubles by reversing the service order within each doubles pair for the final third or fifth set in a match.

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